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Study of Radiation effect on MHD flow past an impulsively started vertical plate with variable temperature and uniform mass diffusion

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Abstract:
This study investigates the effects of thermal radiation on the Magneto hydrodynamic (MHD) flow of a viscous, incompressible, electrically conducting fluid past an impulsively started infinite vertical plate. The physical model accounts for variable surface temperature and uniform mass diffusion in the presence of a transversely applied magnetic field. The dimensionless governing partial differential equations are solved numerically utilizing the Finite Element Method (FEM). At elevated temperatures, the interaction between thermal radiation and buoyancy forces significantly influences the flow field, generating distinct behaviors in both aiding and opposing flow regimes. The individual and combined impacts of key thermo physical parameters—specifically the magnetic field parameter and the buoyancy ratio parameter—on the velocity, temperature, and concentration profiles are systematically analyzed. The findings provide crucial insights into rate-of-cooling optimization for high-temperature manufacturing processes, such as polymer extrusion, metal forming, and liquid metal cooling systems.

Keywords: Magneto hydrodynamics (MHD), Finite Element Method (FEM), Thermal Radiation, Mass Diffusion, Impulsively Started Vertical Plate, Buoyancy Ratio Parameter, Aiding and Opposing Flows

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1.Introduction

The influence of magnetic field on viscous incompressible flow of electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. The rate of cooling can be controlled effectively to achieve final product of desired characteristics by drawing threads, etc. in the presence of an electrically conducting fluid subjected to a magnetic field.

Magneto convection plays an important role in various industrial applications. Examples include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials. MHD finds applications in electromagnetic pumps, controlled fusion research, crystal growing, MHD couples and bearings, plasma jets, chemical synthesis and MHD power generators, etc. in the field of power generation, MHD is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants.

Boundary layer flow on moving horizontal surfaces was studied by Sakiadis [5]. The effect of presence of foreign mass on the free-convection flow past a semi-infinite vertical plate was first studied by Gebhart and Pera [2]. Kumari and Nath [3] studied the development of the asymmetric flow of a viscous electrically conducting fluid in the forward stagnation point region of a two-dimensional body and over a stretching surface with and applied magnetic field, when the external stream or the stretching surface was sent into impulsive motion from the rest. Vajravelu [8] studied the exact solution for hydrodynamic boundary layer flow and heat transfer over a continuous, moving, vertical flat surface with uniform suction and internal heat generation/absorption. In all these studies, the authors have taken the continuous moving surface to be oriented in the horizontal direction. The effect s of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar [6]. MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar et al., [7]. The dimensionless governing equations were solved using Laplace Transform method. R. Muthucumaraswamy and P.Chandrakala [4] studied MHD flow past an impulsively started vertical plate with variable temperature and uniform mass diffusion.

It is proposed to study the radiation effect on MHD flow past an impulsively started infinite vertical plate in the presence of variable temperature and uniform mass diffusion with

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transverse applied magnetic field. The governing equations are solved by the Finite Element Method. When the temperature are high enough, buoyancy effects also generate a significant flow which aids or opposes this induced flow. There are many situations in the manufacturing industry, especially in metal forming and heat treatment. In which one encounters energy transfer to the surroundings, from a moving material. The effect of magnetic field parameter and buoyancy ratio parameter for aiding and opposing flows are studied.

2. Mathematical Formulation.

The Hydro magnetic flow of a viscous incompressible fluid past an impulsively started infinite vertical plate with variable temperature and uniform mass diffusion is studied. Here the x′ -axis is taken along the plate in the vertically upward direction and the y′-axis is taken normal to the plate. It is also assumed that the radiation heat flux is negligible in x-direction as compared to y-direction. Initially, the plate and fluid are the same temperature and concentration. At time t′ > 0, the plate is given an impulsive motion in the vertical direction against gravitational field with constant velocity u_0. The plate temperature is raised linearly with time and the concentration level the plate is raised C_w. A transverse magnetic field of uniform strength B_o is assumed to be applied normal to the plate. The induced magnetic field and viscous dissipation is assumed to be negligible. Then by usual Boussinesq's approximation, the flow is governed be the following equation.

$$\frac{\partial u'}{\partial t'} = g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) + v\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2 u'}{\rho} \qquad \text{.... (1)}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \qquad \text{..... (2)}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial C'}{\partial y'^2} \qquad \text{..... (3)}$$

Where the Rosseland approximation (Brewster [1]) is used, which leads to

$$q_r = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y} \qquad \text{.... (4)}$$

With the following initial and boundary conditions:

$t' \le 0 \quad u' = 0, \quad T' = T'_\infty \; C' = C'_\infty$ for all y'

$t' > 0 \quad u' = u_0, \quad T' = T'_\infty + (T'_w - T'_\infty)At', \; C' = C'_w \; at \quad y' = 0$ …. (5)

$u' = 0, T' \to T'_\infty, C' \to C'_\infty$ As $y' \to \infty$

Where A= $\frac{u_0^2}{\nu}$ (constant).

It is assumed here that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature this is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \qquad \text{………(6)}$$

Using eqn (4) and (.6), eqn (2) gives

$$\rho C_p \frac{\partial T}{\partial t'} = K \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2}$$

On introducing the following non-dimensional quantities

$$u = \frac{u'}{u_0 Gr}, \quad t = \frac{t' u_0^2}{\nu}, \quad y = \frac{u_0 y'}{\nu}, \; \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},$$

$$G_r = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \; G_m = \frac{g\beta^*\nu(C'_w - C'_\infty)}{u_0^3},$$

$P_r = \frac{\mu c_p}{k}$, $S_c = \frac{\nu}{D}$, $M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}$, $N = \frac{Gm}{Gr}$ (buoyancy ratio parameter),

$R = \frac{k^* k}{4\sigma T_0^3}$ (Radiation parameter), where k is thermal conductivity …….. (7)

in equations (1) to (4), we obtains

$$\frac{\partial u}{\partial t} = \theta + NC + \frac{\partial^2 u}{\partial y^2} - Mu \qquad \text{...……. (8)}$$

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$$3RP_r \frac{\partial \theta}{\partial t} = (3R+4)\frac{\partial^2 \theta}{\partial y^2}, \quad \ldots \quad (9)$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}. \quad \ldots \quad (10)$$

With The initial and boundary conditions in dimensionless form are

$$\left.\begin{array}{l} u = 0,\ \theta = 0,\ C=0 \ \text{ for all } y,\ t \le 0 \\ t > 0;\ u = \dfrac{1}{Gr},\ \theta = t, C=1 \ \text{ at } y=0 \\ u = 0,\ \theta \to 0,\ C \to 0 \ \text{ as } y \to \infty \end{array}\right\} \quad (11)$$

All the physical variables are defined in the nomenclature. The equations (8) to (10) subject to the boundary conditions (11) are solved by the Finite Element Method and the solutions are derived as follows:

3. Method of solution

The Finite Element Technique (**Ritz method**) is used to solve equations (8), (9) and (10) subject to initial and boundary condition given in equation (11). To obtain the difference equations, the region is divided into a grid or mesh of lines parallel to y and t axis. Solution of the difference equations is obtained at the intersection of these mesh lines, called nodes. The values of the dependent variables u and T at the nodal points along the planes y=0 are given by u (0, t), T (0, t) and thus are known from the boundary conditions. In the figure (1) h, k are the constant mesh sizes along y and t directions respectively. A scheme is required to find single value at next time level in terms of known values from present time level. We use finite element method to get Crank-Nicolson discretization.

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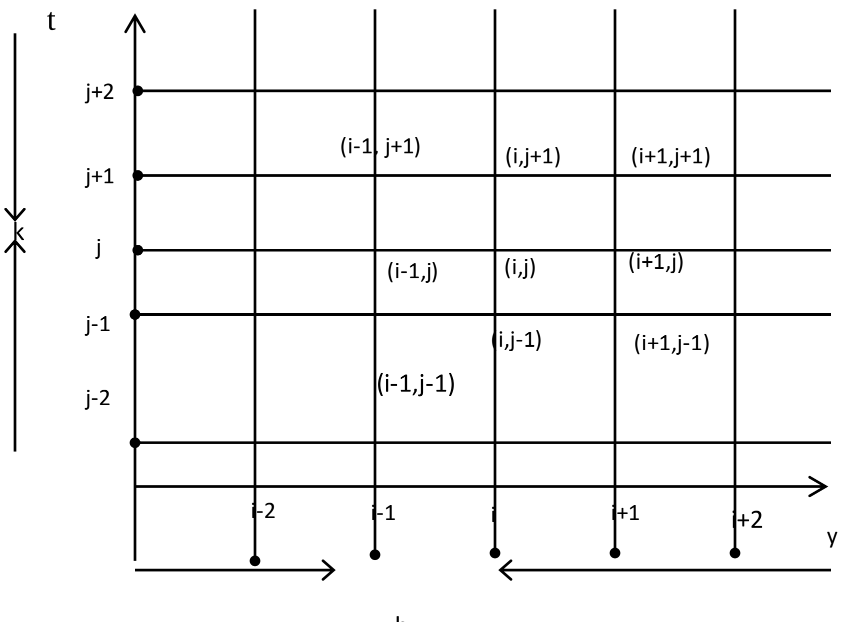

Figure 1.

Consider equation (9)

$$3RP_r \frac{\partial \theta}{\partial t} = (3R+4)\frac{\partial^2 \theta}{\partial y^2}$$

$$\Rightarrow \frac{\partial \theta}{\partial t} = \frac{(3R+4)}{3R}\frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2}.$$

Using finite element method with crank-Nicolson discretization taking h=0.1, k=0.01 therefore r =k / h^2 =1.

The element equation for the typical element (e) $y_j \le y \le y_k$ for the boundary value problem may be written as

$$j^e = 1/2\int_{yj}^{yk} \left[\left(\frac{\partial \theta^e}{\partial y}\right)^2 + 2\theta^e P_r \frac{3R}{(3R+4)}\frac{\partial \theta^e}{\partial t}\right]dy$$

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For the linear piecewise approximate solution

$$\theta^e = N_J(y)\theta_j(t) + N_k(y)\theta_k(t)$$

The element equation is given by

$$j^e = 1/2\int_{y_j}^{y_k}\left\{\left([N_j' \quad N_k']\begin{bmatrix}\theta_J\\ \theta_k\end{bmatrix}\right)^2 + 2P_r\frac{3R}{(3R+4)}[N_J \quad N_K]\begin{bmatrix}\theta_J\\ \theta_k\end{bmatrix}[N_j \quad N_K]\begin{bmatrix}\dot{\theta}_j\\ \dot{\theta}_k\end{bmatrix}\right\}dy$$

$$\frac{\partial j^e}{\partial \phi^e} = \int_{y_j}^{y_k}\left\{\begin{pmatrix}[N_j'N_j' & N_j'N_k']\\ [N_k'N_j' & N_k'N_k'\end{pmatrix}\begin{bmatrix}\theta_j\\ \theta_k\end{bmatrix} + P_r\frac{3R}{(3R+4)}\begin{pmatrix}[N_jN_j & N_jN_k]\\ [N_kN_j & N_kN_k]\end{pmatrix}\begin{bmatrix}\dot{\theta}_j\\ \dot{\theta}_k\end{bmatrix}\right\}dy$$

Where prime denotes differentiation w.r.t. to ‘y’ and dot represent differentiation w.r.t. to‘t’. Here $N_j' = \frac{-1}{h}$; $N_k' = \frac{1}{h}$; where h= $y_k - y_j$; Simplifying, we get

$$\frac{(3R+4)}{3R}\frac{1}{P_r}\frac{1}{h}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}\theta_J\\ \theta_K\end{bmatrix} + \frac{h}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}\dot{\theta}_J\\ \dot{\theta}_K\end{bmatrix} = 0$$

The condition for the extremization of the functional j^e w.r.t the nodal values T_j and T_k are given by

$$\frac{\partial j^e}{\partial \phi^e} = \sum_{e=1}^{n+1}\frac{\partial j^e}{\partial \phi^e} = 0$$

For the elements $y_{i-4} \le y \le y_{i-3}$ and $y_{i-3} \le y \le y_{i-2}$ the assembled equation is given by

$$\frac{1}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 2+2 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}\dot{\theta}_{i-4}\\ \dot{\theta}_{i-3}\\ \dot{\theta}_{i-2}\end{bmatrix} + \frac{(3R+4)}{3R}\frac{1}{P_r}\frac{1}{h^2}\begin{bmatrix}1 & -1 & 0\\ -1 & 1+1 & -1\\ -1 & 1 & 0\end{bmatrix}\begin{bmatrix}\theta_{i-4}\\ \theta_{i-3}\\ \theta_{i-2}\end{bmatrix} = 0 \qquad \ldots\ldots\ldots.(11)$$

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Now put row corresponding to the node i to zero, from Eq. (11) the difference schemes with $l^{(e)}=h$ and Applying the trapezoidal rule, following system of equations in Crank-Nicholson method are obtained:

$$\left(1-\frac{3r}{p_r}\left(\frac{3R+4}{3R}\right)\right)\theta_{i-1}^{n+1}+\left(4+\frac{6r}{p_r}\left(\frac{3R+4}{3R}\right)\right)\theta_{i}^{n+1}+\left(1-\frac{3r}{p_r}\left(\frac{3R+4}{3R}\right)\right)\theta_{i+1}^{n+1}=$$

$$\left(1+\frac{3r}{p_r}\left(\frac{3R+4}{3R}\right)\right)\theta_{i-1}^{n}+\left(4-\frac{6r}{p_r}\left(\frac{3R+4}{3R}\right)\right)\theta_{i-1}^{n}+\left(1+\frac{3r}{p_r}\left(\frac{3R+4}{3R}\right)\right)\theta_{i-1}^{n} \quad \ldots\ldots (12)$$

Now from Eqs (8) and (10), following equations are obtained

$$(1\text{-}3r+\frac{M\ rh^2}{2})U_{i-1}^{n+1}+(4+6r+2rM\ \ h^2)\ U_{i}^{n+1}+(1\text{-}3r+\frac{M\ rh^2}{2})U_{i+1}^{n+1}=$$

$$(1+3r\text{-}\frac{M\,rh^2}{2})U_{i-1}^{n}+(4\text{-}6r\text{-}2rM\ \ h^2\ )U_{i-1}^{n}+(1+3r\text{-}\frac{M\,rh^2}{2})\ U_{i-1}^{n}+6\ \theta k+6\,NC_i^n k \quad ..(13)$$

$$\left(1-\frac{3r}{S_C}\right)C_{i-1}^{n+1}+\left(4+\frac{6r}{S_C}\right)C_{i}^{n+1}+\left(1-\frac{3r}{S_C}\right)C_{i+1}^{n+1}=$$

$$\left(1+\frac{3r}{S_C}\right)C_{i-1}^{n}+\left(4-\frac{6r}{S_C}\right)C_{i-1}^{n}+\left(1+\frac{3r}{S_C}\right)C_{i-1}^{n} \quad \ldots.(14)$$

The solutions of above system of equations (12), (13) and (14) are obtained by using Gauss-Seidel method for temperature, velocity and concentration. For various parameters the results are computed and presented graphically.

4. Results and Discussions:

The numerical values of the velocity and skin-friction are computed for different parameters like magnetic field parameters, G_r = 1, Sc = 2.01, Pr = 7 and buoyancy ratio parameter for aiding (N > 0) and opposing (N < 0) flows, pure heat transfer (N = 0). The purpose of the calculations

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given here is to assess the effect of the parameters M and N upon the nature of the flow and transport.

The velocity profiles for different values of the magnetic field parameter are shown in figure 2. It is observed that the velocity decreases in the presence of magnetic field than its absence. This shows that the increase in the magnetic field parameter leads to fall in the velocity. The effect of buoyancy ratio parameter for both aiding and opposing flows are shown in figure 3. In this case, the velocity decrease in the presence of opposing flows whereas it increases in the aiding flows. From the figure 4 it is observed that the temperature decreases as the radiation parameter increases and from the figure 5 it is observed that concentration decreases with increase of Schmidt number.

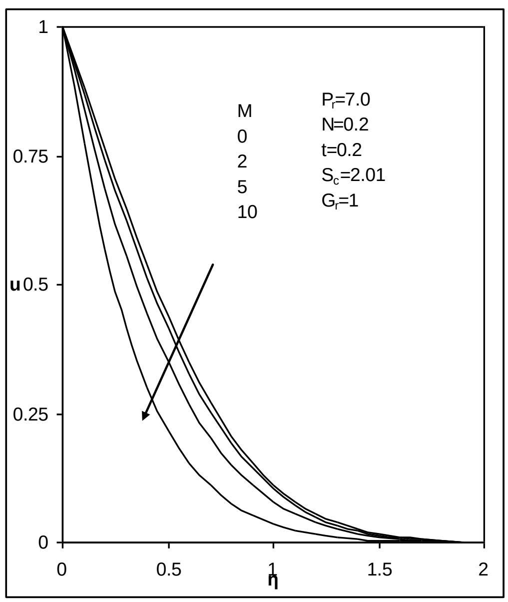

Figure (2.) Velocity profiles for different M.

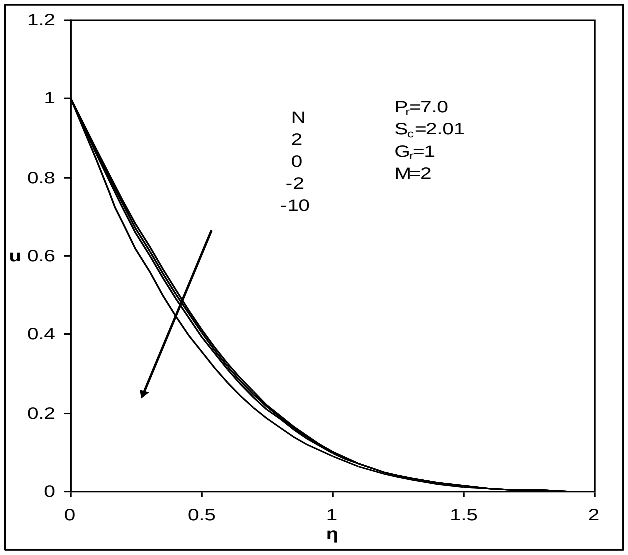

Figure (3.) Velocity profiles for different N

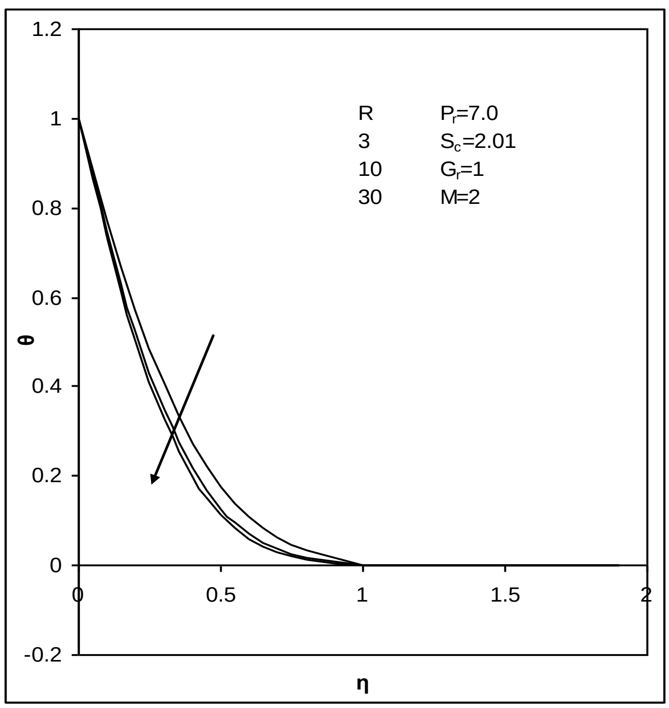

Figure (4) Temperature profile for different R.

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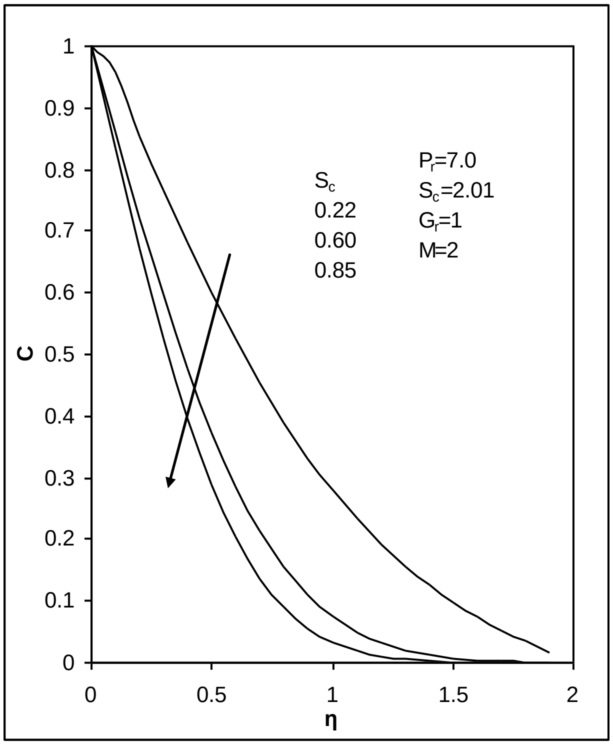

Figure (5) Concentration profiles for different S_c.

From the velocity field the skin-friction is given by

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0}$$ ……… (15) Hence, from the equations (13) and (15), the wall shear stress in the presence of magnetic field is as follows:

M	N	τ
1	2	2.4784
2	2	2.6206
5	2	3.2316

2	0	2.7099
2	-5	2.9333
2	-10	3.1567
2	2	2.6206

Table 1

For G_r=2, P_r=7 and S_c=2.01. It is observed from this table.1, skin-friction increases with increasing values of the magnetic field parameter and also considering Radiation effect. This shows that the wall shear stress increases with increasing magnetic field parameter. It is also observed that the skin-friction increases in the presence of opposing flow and decreases with aiding flows.

5. Conclusions

An analysis is performed to study the flow past an impulsively started infinite vertical plate with variable temperature and uniform mass diffusion in the presence of transverse applied magnetic field. The dimensionless governing equations are solved using finite element method. The effect of magnetic field parameter and buoyancy ratio parameter are studied. The conclusions are as follows:

1. As the radiation parameter increases the temperature decreases.
2. As Schmidt number increases the concentration profile decreases.
3. The velocity decreases in the presence of transverse magnetic field than its absence.
4. The velocity increases in the presence of aiding flows and decreases with Opposing flows.
5. The skin-friction decreases in the presence of aiding flows and increases with opposing flows.
6. Temperature decreases as the Radiation parameter increases.
7. Concentration profile decreases with increase of Schmidt number.

References:

[1] Brewster M Q, Thermal radiative Transfer and properties (John wiley and Sons Inc, New York), 1992.

ISSN: 3048-6726 www.uijmr.in Impact Factor: 6.934 (SJIF) Vol-3,Issue-2 ;April, May & June),2026

[2] Gebhart B., and Pera L., 1971. The Nature of vertical Nature Convection Flows Resulting from the Combined Buoyancy Effects of Thermal and Mass diffusion , Int, J. Heat Mass Transfer,14,2025-2050.
[3] Kumari M., and Nath G., 1999. Development of Two-dimensional Boundary Layer with and applied magnetic field due to an impulsive motion, Indian J, of Pure and Applied Mathematics, 30,695-708.
[4] R. Muthucumaraswamy and P.Chandrakala. 2003. MHD flow past an impulsively started vertical plate with variable temperature and uniform mass diffusion, Journal of Energy, Heat and Mass transfer, vol 25, pp 243-251.
[5] Sakiadis B.C., 1961. Boundary layer behavior on continuous solid surface: II Boundary layer on a continuous Flat surface, AIChE journal, 7,221-225.
[6] Soundalgekar V.M., Gupta S.K., and A Ranake R.N., 1979. Free convection effects on MHD stokes problem for a vertical plate, Nuclear Engg. Des., 51,403-407.
[7] Soundalgekar V.M., Patil P.R. and jahagirdar M.D., 1981.MHD Stokes problem for a Vertical plate with variable temperature, Nuclear Engg.Des., 64, 39-42.
[8] Vajravelu K., 1988. Hydromagnetic Convection at a continuous moving surface, Acta Mechanica, 72,223-232.